Part I: Platonic Solids *
Relations among them, Simple Truncations
See Part II below: Archimedean Solids

There are five (and only five) Platonic solids. Three of them are easy to imagine—the Cube, Octahedron and Tetrahedron—while the remaining two are more difficult: the Icosahedron and Dodecahedron. The earliest known models date from the Stone Age.

What to do in 3D-XplorMath?

First, the program shows how the other four Platonic solids are obtained from the Cube: Select first one of the other polyhedra, then in the Action Menu:

`Show Relation with Cube`.

For the Octahedron one sees that its six vertices are the midpoints of the faces of a cube; the Octahedron faces are equilateral triangles. The Tetrahedron sits in the cube so that its four vertices are vertices of the Cube, and the six Tetrahedron edges are face diagonals of the Cube. The Icosahedron can be placed

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* This file is from the 3D-XplorMath project. Please see: http://3D-XplorMath.org/

inside a Cube so that its twelve vertices lie on the six faces of the Cube: see the default morph in 3DXM, preferably when `Show Relation with Cube` is chosen. The Dodecahedron can be placed around a Cube so that its twelve pentagon faces rest on the twelve edges of the cube. See again the default morph going from the Rhombic Dodecahedron via the Platonic Dodecahedron to a cube with subdivided faces.

Second, by cutting off appropriately the vertices or the edges of a Platonic solid one obtains the simpler **Archimedean solids**. These are polyhedra whose faces are (several kinds of) regular polygons, whose edges all have the same length and whose vertices all look the same. Select in the Action Menu any of the three `Truncations` and then do the associated default Morph. For example, the `Edge Truncations` morph from one Platonic solid to another one (which is called its dual).

Third, since our intuition handles two dimensions better than three, it is interesting to project the Platonic solids from their midpoints onto a circumscribed sphere. View in Wire Frame and select in the Action Menu: `Show Central Projection to Sphere`.

These two-dimensional spherical views of the Platonic solids come very close to explaining why there are no other such beautiful polyhedra.

Fourth, the Icosahedron and the Dodecahedron have very beautiful Stellations, polyhedra that fascinated Kepler. Select (Action Menu): `Create Stellated`. Note that all the mentioned views have their own default morphs. Kepler imagined the stellated Dodecahedron as having pentagon stars as faces.
In addition, one can select in the Polyhedra Menu `Kepler's Great Dodecahedron`. Kepler viewed its faces as Pentagons, the vertices being those of an Icosahedron. In 3DXM it is drawn as a (negative) stellation of the Icosahedron.

Fifth, for the Cube and the Icosahedron there are two special entries in the Action Menu when viewing these solids in Wire Frame. For the Cube select `Show Intersection With Plane`, preferably in one of the stereo modes. The plane is represented by random dots and the dots inside the Cube are deleted; the Cube can be rotated and moved forward and backward, always showing its polygonal intersection with the plane. For the Icosahedron select in the Action

Menu `Add Borromean Link`, preferably in one of the stereo modes. Note how the boundaries of the emphasized rectangles are intertwined or linked. The edge lengths of each rectangle are equal to the lengths of an edge and a diagonal of a regular pentagon, thus showing the relation of the Icosahedron inside the Cube with the Golden Ratio. Also, the default morph of this image is worth viewing.

Finally, stone objects with Platonic Symmetry were found, mainly in Scotland. They were dated 2500 B.C. They are carved from bigger stones, but they look as if they were conceived as collections of balls. Therefore we have added the Action Menu entry: `Show As Stone Balls`. In Patch Display the balls are fine triangulations of the Bucky Ball, in Wire Frame the balls are shown with random dots. One can view other sphere triangulations after one has selected (in Patch Display) `Create Subdivided`: another entry appears: `Triangulate Further`.

Part II: Archimedean Solids

Here is the definition again: All faces are regular polygons (of up to three different kinds). All edges have the same length. All vertices (with their outgoing

edges) are congruent. In addition to the five Platonic solids there are twelve of them.
We have already seen the simplest ones: Truncate the vertices of a Platonic solid; there are two possibilities, if some portion of the edges remain this is called `Standard Truncation`, and if the truncation cuts go through the midpoints of the edges we have a `Midpoint Truncation`. One can also `truncate the edges`; this deformation leads to the same Archimedean solid if one starts from the Octahedron or the Cube, and also if one starts from the Icosahedron or the Dodecahedron. Two more are obtained if one truncates the edges and the vertices; the deformation is easier to observe if one uses the standard truncation on either the Cubeoctahedron or the Icosidodecahedron. In fact, not quite the standard truncation, because that would make rectangles instead of squares from the truncated vertices.
Finally there are the `Snub Polyhedra`. We could not find what 'snub' means in this context. We describe the construction and call it 'to snub'. Each face of a Platonic solid is scaled down from its midpoint and also rotated around the midpoint. The 'snubbed' polyhedron is the convex hull of these deformed faces.

This 2-parameter deformation can be adjusted to give a 1-parameter family of polyhedra whose faces are either regular polygons or isosceles triangles. In each family is an Archimedean solid. A snubbed Tetrahedron is an Icosahedron, snubbed Cube and Octahedron give the same Archimedean solid and also snubbed Dodecahedron and Icosahedron agree.

One can probably understand all these truncations better if one selects in the Action Menu

`Snub Or Truncate Polyhedron In Polyhedron`.

This will add the original polyhedron (as wire frame) to the truncation.

Note also that each selection in the Action Menu will cause that the default deformation, `Morph` in the Animation Menu, is adjusted to the Action Menu selection.

H.K.