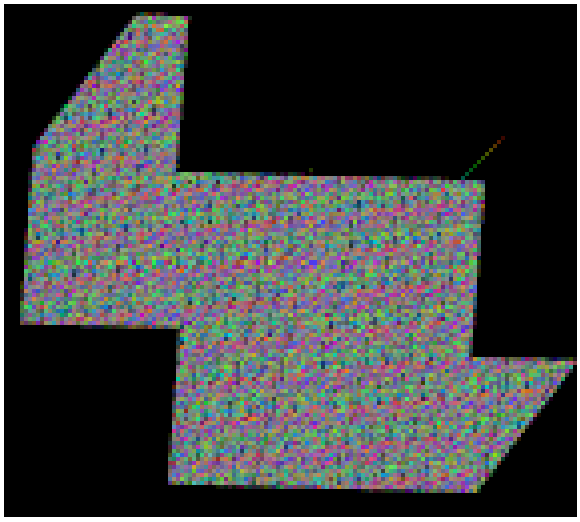
About Welling Iteration *



1. Welling Maps and Welling Iterations

A *Welling Map* is certain kind of map $W : \mathbb{R}^n \to \mathbb{R}^n$. In more detail, a Welling Map depends on a positive integer $N \geq n+1$ and N points $P_1, \ldots, P_N$ in $\mathbb{R}^n$. Given N and the P_i and $x \in \mathbb{R}^n$, $W(x) \in \mathbb{R}^n$ is defined by the following:

`Welling Map Algorithm`

1. `Compute the` N `dot-products`
   $\texttt{dot}_i :=< x, P_i >,\ i = 1, \ldots, N$.
2. `Let` j `be the first` i `such that` $\texttt{dot}_i$ `maximizes these dot-products.`
3. `Define` $W(x) = x - P_j$.

In `3D-XplorMath` $n = 2$ and the integer N is given by the parameter `ii`, and is initially equal to 5. It can be changed by selecting

*This file is from the 3D-XplorMath project. Please see: http://3D-XplorMath.org/

the first item of the Settings Menu. The two coordinates of each of the points P_i are initially set automatically by a random number generator. Each time the item `Next Fractal` is chosen from the Action menu, the coordinates of the P_i are randomized again, producing a new Welling Map. The random number generator is always initialized the same way when `Welling Iteration` is selected from the Fractal and Chaos Menu, so that a sequence of experiments done with `3D-XplorMath` is repeatable. **However** if `Random Fractal` is chosen from the Action Menu, then the random number generator is first randomly initiated and then a new set of P_i are chosen, allowing serendipity in the choice of Welling Map if desired.

A *Welling Iteration* refers to the discrete dynamical system obtained by iterating some fixed Welling Map. It is believed that for appropriate choices of the Welling Map, the Welling Iteration has interesting properties, such as a fractal attractor. The details are under active investigation and a little of what is known and conjectured will be discussed below. The Welling Iteration was programmed into `3D-XplorMath` to make it possible to visualize properties of Welling Iterations and thereby, hopefully, help to better understand their structure.

2. What is Displayed

As soon as `Welling Iteration` is selected from the Fractal and Chaos menu (or `Next Fractal` or `Random Fractal` is selected from the Action menu) a certain array of points equi-spaced along a line segment is iterated. The original collection of points is drawn

on the screen, each in a different color, the next hundred iterations of the points are ignored, and then after that each iteration of the array is plotted (to an offscreen bitmap for speed) and every `ee` iterations the offscreen bitmap is copied to the screen. In each iteration, each point of the array is plotted in its original color. The result is that one sees on the screen the original line of colored points and a gradually growing collection of iterations of these points. The interesting parts of this collection quickly become to dense to see well what is going on. If one clicks the mouse on the window, then the iteration will cease. At this point one can draw a small rectangle on the screen by first holding down Command, then dragging out the rectangle with the mouse, then releasing Command and finally releasing the mouse. The small rectangle dragged out will zoom to fill the screen, and the iteration will start over (NOT continue) so you will be able to see what is happening in that rectangle in more detail. This can be repeated as many times as desired.

3. Parameter Initialization

- Number of Iterations = 9000000;
- xmin = - 1.8; xmax = 1.8; ymin = - 1.1; ymax = 1.1;
- xCenter := (xmin+xmax)/2; yCenter := (ymin+ymax)/2;
- aa = 1.0; bb = 1.0;
  (aa, bb) are coordinates of one end of interval of iterated points.

- cc = 0.5; dd = 0.5;
  (cc, dd) are coordinates of other end of interval of iterated points.
- tResolution = 150;
  The number of points interpolated between the above endpoints.
- ee = 100; For speed, during curve iteration, drawing is done to an offscreen bitmap, and the offscreen bitmap is copied to screen every ee interations.
- ff = 16; number of points in mouse selected interval
- gg = 0; unused
- hh = 0; unused
- ii = 5; Number of WellingPoints

M.W. and R.S.P.