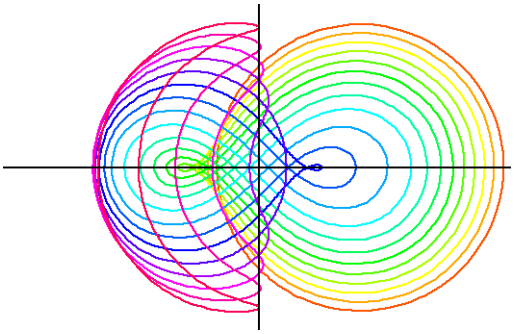
About Mechanically Generated Curves *

Examples

Presently we have the following mechanically generated plane curves programmed together with a decoration which shows this generation and the corresponding construction of the tangents of the curve:



Epi- and Hypocycloids,
all other rolling curves.
Also: Tractrix, Cissoid,
Conchoid, Lemniscate.

This image is obtained with `Color Morph` in the Animation menu, it shows the family obtained from the current drawing mechanism (here Lemniscate).

Moving Planes

It is often convenient to discuss such mechanical generations in terms of two planes, a fixed plane on which the drawing is done (paper plane) and a second plane which is attached to that piece of the mechanical con-

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* This file is from the 3D-XplorMath project. Please see:

http://3D-XplorMath.org/

traption that holds the drawing pen (drawing plane). In the case of rolling curves we have the drawing plane attached to the rolling wheel, in the case of the Lemniscate the drawing plane is attached to the middle one of the three connected moving segments.

> *We think of the orbits of the points of the drawing plane as curves that are mechanically generated by the apparatus under consideration.*

The velocity vectors of these orbits clearly give a time dependent vector field. Since this vector field is obtained by differentiating the orbits of a family of **isometries** we obtain at each time t the vector field of a Euclidean **group of motions**, in other words: for most t the vector field consists of the velocity vectors of a rotation, a rotation around the so called momentary center of rotation. This way of looking at the generation gives immediately tangent constructions for all orbits: join the momentary center of rotation to the moving point, the perpendicular line through the point is tangent to its orbit.

It is therefore useful to visualize the movement of the drawing plane together with the time dependent velocity field of its points. We have done this by decorat-

ing the drawing plane with not too many but enough random points so that the movement of the drawing plane becomes visible, but the curve under consideration is not obscured. Moreover, to make the vector field visible at each moment t, we have drawn the random points not once, but at two subsequent positions. This picture is interpreted by the brain correctly.
Finally, one has to determine the momentary center of rotation. This is different for each construction. For rolling curves the definition of "rolling" is such that that point, where the rolling wheel touches the fixed curve ("street"), is the momentary center of rotation. In general one has to look for points of the mechanical apparatus for which the direction of the momentary movement ("orbit tangent") can be decided. The momentary center is then on the line ("radius") perpendicular to the tangent, so that two such lines are needed. The 3DXM demos use green lines to determine the momentary center.

H.K.