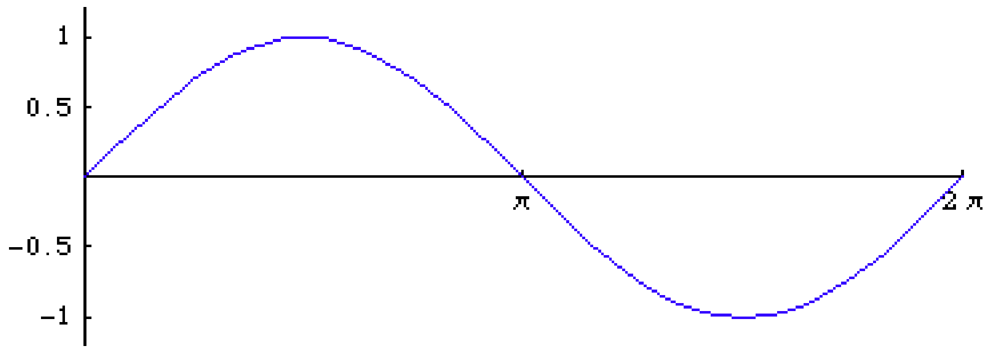
Sine [*]



All of the so-called trignometric or circular functions: sine, cosine, tangent, secant, cosecant, cotangent can be defined in terms of the single function sine. Its graph is often referred to as a Sinusoid (or as a sinusoidal curve).

The sine function has been associated with trignometry, since the beginnings of civilization and was an important tool in metrology, the science of measurement. When the function concept, calculus, and analytic geometry were introduced arond 1700, the sine function became divorced from its relation to triangles—it appears unexpectedly throughout analysis. One reason is that it captures well the concept of a periodic wave, a fundamental concept in physics. But perhaps even more importantly, so-called Fourier analysis gives a prescription for representing any periodic functions as an (infinite) linear combination of sines and cosines. It is nearly impossible to exaggerate the profound effect of this idea in the development of modern pure and applied mathematics.

[*] This file is from the 3D-XploreMath project.
Please see http://rsp.math.brandeis.edu/3D-XplorMath/index.html

Excerpt from Robert C. Yates (1974):

> Trigonometry seems to have been developed, with certain traces of Indian influence, first by the Arabs about 800 as an aid to the solution of astronomical problems. From them the knowledge probably passed to the Greeks. Johann Müller (c.1464) wrote the first treatise: *De triangulis omnimodis*; this was followed closely by others.

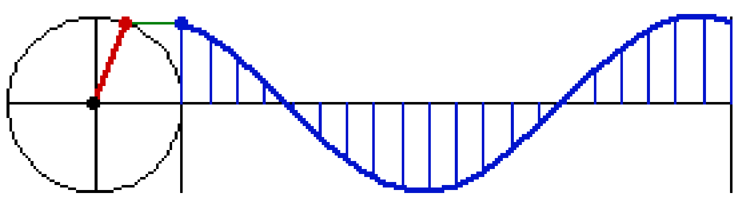
The reason behind the terminology "circular function" is as follows: as a point moves around a circle, its height is the value of sine of the distance it has moved along the circle. Here is a step by step description:

1. Draw a circle of radius one centered on the origin.

2. Let P be any point on the circle.

3. Let θ be the radian measure of the angle (1,0), (0,0), P, i.e., the length of the segment of arc from $(0, 1)$ to P.

4. Then the coordinates of the point P are $(\cos(\theta), \sin(\theta))$.



If we limit θ to the interval $(0, \pi/2)$, the above becomes the classical definition of the sine of an acute angle of a right triangle as "the ratio of the side opposite to the hypotenuse".

The other trig functions can be defined in terms of sine, as follows.

$$
\begin{aligned}
\csc(\theta) &= 1/\sin(\theta) \\
\cos(\theta) &= \sin(\theta + \pi/2) \\
\sec(\theta) &= 1/\cos(\theta) \\
\tan(\theta) &= \sin(\theta)/\cos(\theta) \\
\cot(\theta) &= 1/\tan(\theta)
\end{aligned}
$$

If θ is an angle in standard position, we have the following formulas, where r is the length of the hypotenuse.

$$
\begin{aligned}
\sin(\theta) &= y/r \\
\cos(\theta) &= x/r \\
\tan(\theta) &= y/x
\end{aligned}
$$

Here are some other common identities that are less obvious:

Pythagorean

$$\sin(x)^2 + \cos(x)^2 = 1$$

Sum

$$\begin{aligned}
\sin(a) + \sin(b) &= 2\sin((a+b)/2)\cos((a-b)/2) \\
\sin(a) - \sin(b) &= 2\cos((a+b)/2)\sin((a-b)/2) \\
\cos(a) + \cos(b) &= 2\cos((a+b)/2)\cos((a-b)/2) \\
\cos(a) - \cos(b) &= -2\sin((a+b)/2)\sin((a-b)/2)
\end{aligned}$$

Addition, subtraction, Double-angle

$$\begin{aligned}
\sin(a+b) &= \cos(a)\sin(b) + \cos(b)\sin(a) \\
\cos(a+b) &= \cos(a)\cos(b) - \sin(a)\sin(b) \\
\tan(a+b) &= (\tan(a) + \tan(b))/(1 - \tan(a)\tan(b))
\end{aligned}$$

Product

$$\begin{aligned}
\sin(a)\cos(b) &= (\sin(a+b) + \sin(a-b))/2 \\
\cos(a)\cos(b) &= (\cos(a+b) + \cos(a-b))/2 \\
\sin(a)\sin(b) &= (\cos(a-b) - \cos(a+b))/2
\end{aligned}$$

Half-angle (Sign must be chosen)

$$\begin{aligned}
\sin(x/2) &= \pm\sqrt{((1-\cos(x))/2} \\
\cos(x/2) &= \pm\sqrt{(1+\cos(x))/2} \\
\tan(x/2) &= \pm\sqrt{(1-\cos(x))/(1+\cos(x))} \\
&= \sin(x)/(1+\cos(x)) \\
&= (1-\cos(x))/\sin(x)
\end{aligned}$$

Inverse Identities

$$\begin{aligned}
\arcsin(-x) &= -\arcsin(x) \\
\arccos(x) + \arccos(-x) &= \pi \\
\arcsin(x) + \arccos(x) &= \pi/2 \\
\arcsin(x) &= \operatorname{arccsc}(1/x) \\
\arccos(x) &= \operatorname{arcsec}(1/x) \\
\arctan(x) &= \operatorname{arccot}(1/x)
\end{aligned}$$

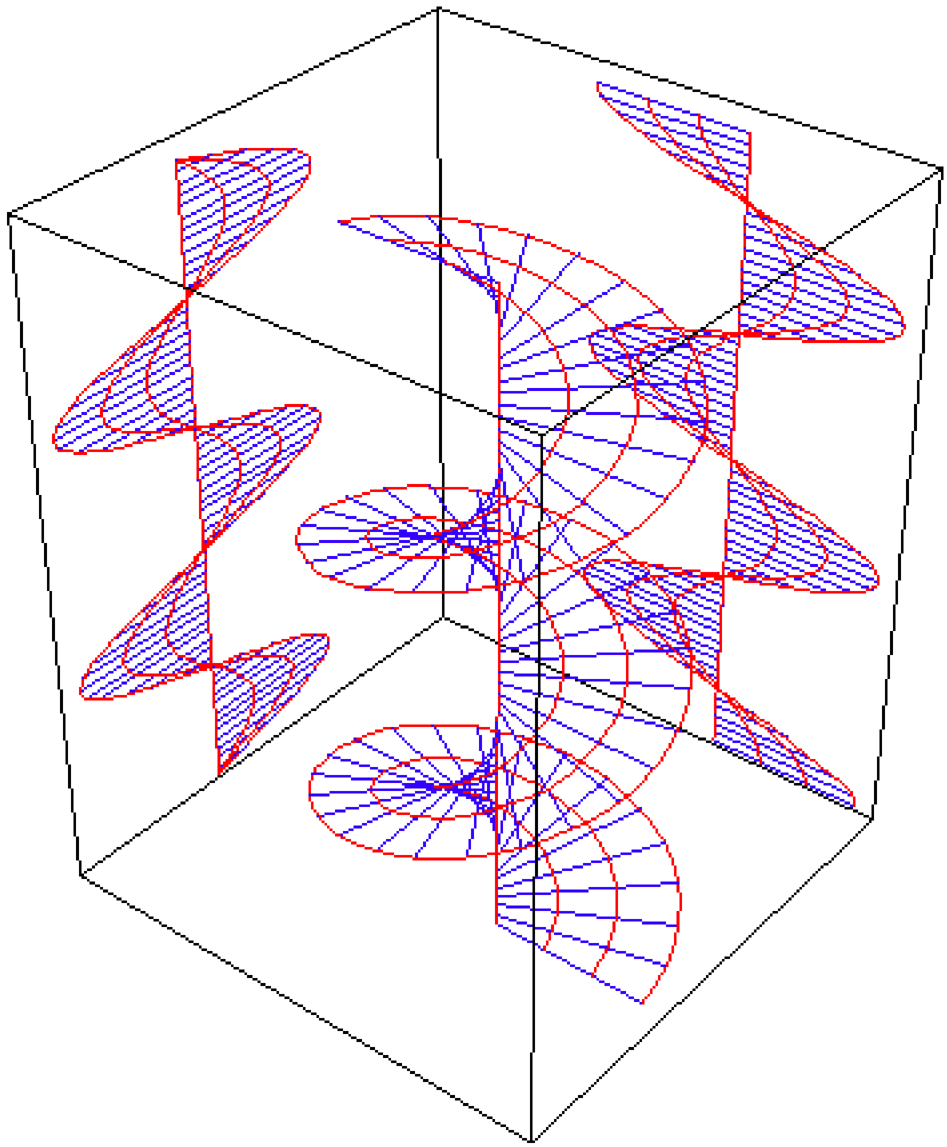
Law of Sines and Law of Cosines

For any triangle with side lengths a, b, and c, whose opposite angles are α, β, and γ respectively,:

$$\sin(\alpha)/a = \sin(\beta)/b = \sin(\gamma)/c$$
$$a^2 = b^2 + c^2 - 2bc\cos(\alpha)$$

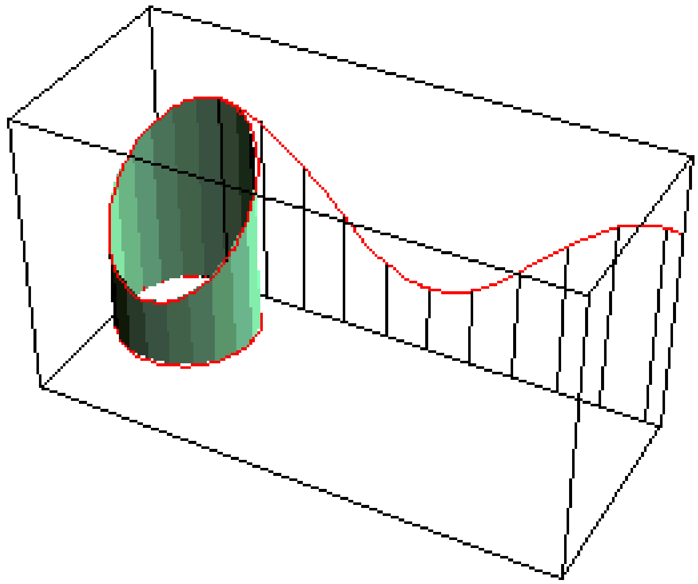
1 Trivia

1.1 Orthogonal Projection of Helix



The sinusoid is the orthogonal projection of the helix space curve. In 3DXM, helix can be seen under the Space Curves category.

1.2 The development of a cut cylinder



The sinusoid is the development of an obliquely cut right circular cylinder—i.e., the edge of the cylinder rolls out to a sinusoid.

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