About Catenoid Enneper

H. Karcher

The surfaces Wavy Enneper, Catenoid Enneper, Planar Enneper, and Double Enneper are finite total curvature minimal immersions of the once or twice punctured sphere—shown with standard polar coordinates. These surfaces illustrate how the different types of ends can be combined in a simple way.

Here an Enneper perturbation with ee tongues (size adjusted with aa) crumples one rim of a catenoid in the suggested morphing. It is also interesting to choose aa=0.65, ee=13, or so. The catenoid is aa=0.

Gauss map : $Gauss(z) = \big(4 - aa(1 + z^{ee})\big)/z$
Differential: $dh = (1 + aa^2/2) \cdot Gauss(z)\, dz$.

The pure Enneper surfaces (Gauss(z) = z^k) and the Planar Enneper surfaces have been re-discovered many times, because the members of the associate family are *congruent* surfaces (as can be seen in an associate family morphing) and the Weierstrass integrals integrate to polynomial (respectively) rational immersions. Double Enneper was one of the early examples in which I joined two classical surfaces by a handle; we suggest to morph the size of the handle or the rotational position of the top Enneper surface against the bottom Enneper surface. Formulas are taken from:

H. Karcher, Construction of minimal surfaces, in "Surveys in Geometry", Univ. of Tokyo, 1989, and Lecture Notes No. 12, SFB 256, Bonn, 1989, pp. 1–96.

For a discussion of techniques for creating minimal surfaces with various qualitative features by appropriate choices of Weierstrass data, see either [KWH], or pages 192–217 of [DHKW].

[KWH] H. Karcher, F. Wei, and D. Hoffman, The genus one helicoid, and the minimal surfaces that led to its discovery, in "Global Analysis in Modern Mathematics, A Symposium in Honor of Richard Palais' Sixtieth Birthday", K. Uhlenbeck Editor, Publish or Perish Press, 1993

[DHKW] U. Dierkes, S. Hildebrand, A. Kuster, and O. Wohlrab, Minimal Surfaces I, Grundlehren der math. Wiss. v. 295 Springer-Verlag, 1991