About Kusner's Dihedral Symmetry Surface

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Pseudo-Code for "Kusner's Dihedral Symmetry Surfaces":

Let $p = \max(\text{Round}(ee), 2)$
and let

$\Re(z) = u\cos(v)$
$\Im(z) = u\sin(v)$
Then $P(z) = \Re(a(z)V(z)) + (0, 0, aa)$
where $a(z)$ is the complex number

$$a(z) = \frac{1}{(z^p - z^{-p} + (2/(p-1))\sqrt{(2p-1)})}$$

and $V(z)$ is the complex vector

$$V(z) = (i(z^{p-1} + z^{1-p}), z^{p-1} + z^{1-p}, (i(p-1)/p)(z^p + z^{-p}))$$

This is a minimal surface with dihedral symmetry of order $2p$ if p is odd and $4p$ if p is even. The default value of ee is 4. This gives the inversion in the unit sphere of the "Morin Sphere Eversion Midpoint" Willmore surface (see the surface surface menu). On the other hand, when ee = 3 this gives the Inverted Boy's Surface (on this menu).
For full details, see:

R. Kusner, Conformal Geometry and Complete Minimal Surfaces, Bulletin of the AMS, v.17, Number 2, October 1987, pp291–295.