The Logarithmic Spiral

The parametric equations for the Logarithmic Spiral are:

$$x(t) = aa * \exp(bbt)\cos(t)$$
$$y(t) = aa * \ exp(bbt)\sin(t)$$

This spiral is connected with the complex exponential as follows:

$$x(t) + i\,y(t) = aa\exp((bb + i)t).$$

The animation that is automatically displayed when you select Logarithmic Spiral from the Plane Curves menu shows the osculating circles of the spiral. This illustrates an interesting theorem, namely if the curvature is a monotone function along a segment of a plane curve, then the osculating circles are nested. (See page 31 of J.J. Stoker's “Differential Geometry”, Wiley-Interscience, 1969).

For the logarithmic spiral this implies that the plane minus the origin is foliated by its osculating circles.

WHAT!? Hey! Wait a minute! If a smooth manifold M of dimension n is foliated by leaves of dimension k, and if S is a k-dimensional connected submanifold of M such that S is tangent at every point to one of the leaves, then S is an open subset of a leaf. But taking M to be the punctured plane, and S the logarithmic spiral, the osculating circle foliation gives a counterexample to this well-known theorem (which is little more than the definition of a leaf). This paradox was pointed out to me by Étienne Ghys. (Read words backwards below for the explanation.)

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